(Anti)symmetric ornaments in math lessons

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Abstract

Mathematics is difficult science for many students, but teacher's obligation is to use many interesting ways in order to transfer mathematical knowledge, to enable students realize that mathematics is omnipresent, to encourage its applying and to teach students to analize the problems. Taking into account all aspects, it dawned upon me that I might use (anti)symmetric ornaments and their derivation from few prototiles - OpTiles in mathematical lessons. The main aim of this study is to make students realize that dealing with math is like Jigsaw puzzle – by using simple elements, you can make complex structures, while solving mathematical problem means combining some concrete basic knowledge for a particular area. Ornaments, which are created by students, are interesting for detailed study from mathematical point of view, for creating mathematical tasks by students and by teachers and for applying on clothes, jewellery and other decorative things. In this way I also encourage students to explore areas of mathematics, art and their application.

1 Introduction

Besides the aim of teaching mathematics at grammar schools is adopting mathematical knowledge by students, the aim is also contributing to all-round development of students' personalities. Students should develop awareness on universality and the application of mathematical knowledge and the way of thinking. Very important demand in the process of teaching mathematics is to make students possible to learn it mainly during the classes. This can be accomplished only if all the students participate actively in the teaching process. Combining frontal work with group work is necessary for students to get more knowledge and to increase self-esteem, motivation, willingness to work and creativity.
An important part of teaching mathematics is visualization with its aim to make understanding of mathematics much easier. The aim is to show some mathematical structures and their characteristics in drawings and animations. Art can be enriched in that way and vice versa.
For this purpose I used optical ornaments at math lessons. Optical ornaments might help students to understand some mathematical lessons more easily and make it more interesting. Students will be in opportunity to combine mathematics and art, create new tasks and in that way they will better understand some

of mathematics areas. Inspired by the beauty of optical ornaments, we started applying ornaments on some decorative objects, on clothes and jewellery.

2 Modularity in art

Regarding the modularity we considered the use of several basic elements (modules) for constructing a large collection of different (modular) structures. What is modularity principle in science and art?
Accordingly to [1], "in science, the modularity principle is represented by search for basic elements (e.g. elementary particles, prototiles for different geometric structures, etc). In art, different modules (e.g. bricks in architecture or in ornamental brickwork, etc) occur as the basis for modular structures. In various fields of (discrete) mathematics, the important problem is the recognition of some set of basic elements, construction rules and an (exhaustive) derivation of different generated structures...In a general sense, the modularity principle is a manifestation of the universal principle of economy in nature: the possibility for diversity and variability of structures, resulting from some (finite and very restricted) set of basic elements by their recombination. In all such cases, the most important step is the first choice (recognition or discovery) of basic elements. This could be shown by examples from ornamental art, where some elements originating from Paleolithic or Neolithic art are present till now..."
In many cases, the derivation of discrete modular structures is based on symmetry. Using the theory of symmetry and its generalizations (simple and multiple antisymmetry, coloured symmetry, etc) for certain structures it is possible to define exhaustive derivation algorithms, and even to obtain some combinatorial formula for their enumeration.

As the example of modular structures lying on the border between the art and mathematics could be considered:
- The set of modular elements for derivation of possible and impossible objects "SpaceTiles";
- Different knot projections occurring in knot designs (Islamic, Celtic, etc) derived from the regular and uniform plane tessellations by using few basic elements "KnotTiles";
- Antisymmetry ornaments and their derivation from few prototiles – "OpTiles";

If we consider the Paleolithic key-patterns, Celtic ornaments and op-art works, we can see their joint basis: basic (anti)symmetric prototiles obtained by a division of a rectangle with their diagonal lines into two antisymmetric (complementary) prototiles, where one of them or both are used. This can also explain somewhat hesitating visual impression that such patterns produce: the constant effect of flickering, when the eye recognizes black and white pattern and oscillates between them. From "black-white" prototiles we can obtain the corresponding black-white patterns. The series of such tilings derived from the four prototiles is represented by "OpTiles". Most of antisymmetric ornaments could be derived by recombination of few basic "OpTiles". If we try to explain what we see as one object, we come to the conclusion that we actually have a variety of objects to choose from. From this infinity, our perception most often opt for one "natural" (or the simplest possible) interpretation.

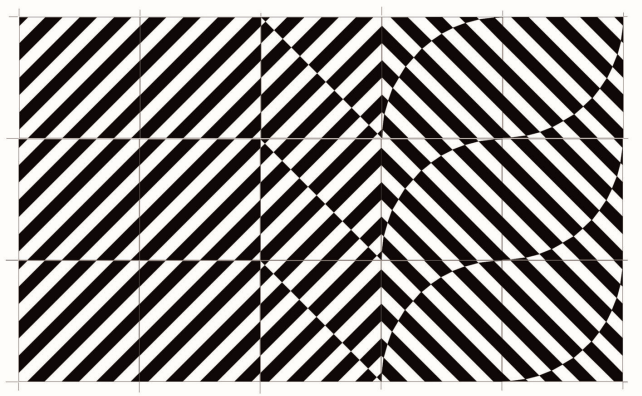

Figure 1: Paper with basic elements.

3 OpTiles in mathematical classes

The aim of combination mathematics and art is to show students that mathematics is all around us and make them create positive attitude towards mathematics in general.
Group work is essential for this purpose. By using optical ornaments in class, students will realize a huge application of mathematics in art which was present in distant past as well.
During these lessons, students familiarise themselves with link between mathematics and art and with the notion and types of modularity in art.

3.1 Creating magical ornaments

Using slide presentation with needed information, pictures and videos which are about connecting mathematics and art, teacher should motivate students for work afterwards. After lecture about modularity, students are divided into groups of 4 or 5 members (at first students might split into groups as they like and afterwards according to different criteria). Students should be explained the method of work.
Each group of students has sheets of paper with basic elements drawn (Figure 1), scissors, glue and white piece of paper. Students cut out basic elements and get black-white squares which can use to make complex structures.
We can notice 5 different squares (Figure 2) on the paper. Students are free to use the square they want, and how many they want. They can put the squares side by side, an edge by an edge, but we get more interesting combination by moving the squares. Students realize very quickly that they can continue placing a black line onto the another one, and a white one onto the another white, so the most interesting effects are accomplished when at a moment the line is cut off by an opposite colour.
In the end, they stick their structures onto a white piece of paper. Teachers supervise the work and give students some additional instructions and explanations, at the same time they encourage team work. Due to the beauty of optical ornaments and the impression they make on every individual, students are eager to participate in the work.

Figure 2: Basic elements.

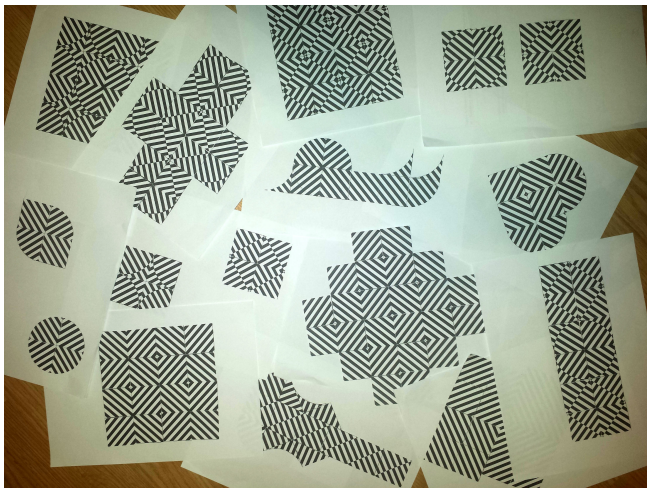

Figure 3: Some of students' works.

At first, students choose to do one creative assignment within the group since they feel more comfortable that way, but later on they come to realize that making more complex structures is not so hard and they plunge into making their own ornaments. At this moment, their self-esteem is boosted and they become more creative. Even those students who are not so good at mathematics realize they could do the task on their own.
Mathematics is not a bogey anymore. It is imperative that these lessons should be organized during double period so there is enough time for all activities. At the end of the class, members of each group show their work, compare notes and talk about visual perception and the particular feelings these ornaments inspire in them. They also talk about where mathematics is present in all those works (symmetry, asymmetry, rotation, etc) and about application of these optical ornaments in real life (clothes, jewellery, etc). After the lesson, students take pride in their works and boast about them. Some of students' works are shown in the Figure 3.

3.2 Applying magical ornaments in mathematical tasks

Using optical ornaments we develop quality group work in the classroom, encourage students to explore the field of mathematics and art, but ornaments that the students created are very favorable for a more detailed study from the mathematical point of view.
Teachers can organize classes like these when they need to explain notions symmetry, asymmetry and antisymmetry.

Figure 4: Some of the ornaments which we can use to create tasks.

When it comes to order the study of complex figure, the tasks that the students can do may be just ornaments that they created (Figure 4). This is a good way for them to better understand geometry and mathematics.
Ornaments that the students created teacher can process in one of the processing programs for drawing and print them on paper in precise dimensions. So in that way students can clearly see the parts and determine the dimensions of ornaments - shapes and they can solve a lot of tasks.
All students successfully calculate circumferences and areas of these figures. We can create a lot of new tasks on this topic and we can also increase requirements. Students can also create new tasks on this topic. It is very interesting because they are rarely in possibility to create tasks for their friends from the same class. They can create new ornaments and tasks at home using basic elements from paper or using computer.
Solving mathematical tasks is now the same as doing jigsaw puzzle - by using simple elements, you can make more complex structures, while doing some mathematics exercise means combining some concrete basic knowledge for a particular field.
In the Figure 4 we can see some of the ornaments which we can use to create tasks, for example for calculating circumference and area of the interesting ornaments.

Also, when students understand and apply percentage, teacher can also use ornaments which students created earlier. Teacher can create different tasks with different requests. Students, which are not talented for mathematics, can calculate percentage on simpler tasks than other students, just using counting the parts of simple ornaments. All students will understand percentage on different levels.
For example, I will show one tasks on this topic:

On the paper (Figure 5), which shape is square, there is black-white flower. The flower touches paper's edges. Anne wants to cut out the flower using scissors and stick it on her notebook. Percentage of paper which Anne should remove is approximately equal to:

a) 35% b) 30% c) 25% d) 23% e) 20%

Solution:
Final solution is: 30%.

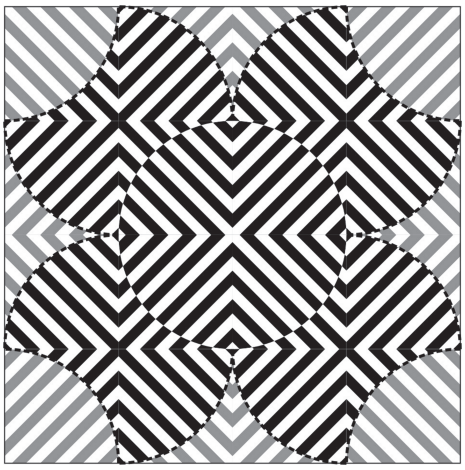

Figure 5: Optical flower.

More details:

The area of the whole paper (the square whose length edge is a): $P_1 = a^2$.

Having remodelling the parts, which are outside the flower, it may be brought to a conclusion that the area of that part of paper may be calculated as a sum of area of a circle with a radius $\frac{a}{4}$ and area of parts which we get by dropping the circle with a radius $\frac{a}{4}$ from the square whose edge length is $\frac{a}{2}$.

The area of the paper, which needs to be removed (outside the flower):

$$P_2 = \frac{a^2\pi}{16} + 2(\frac{a^2}{4} - \frac{a^2\pi}{16}) = \frac{a^2(8-\pi)}{16}$$

The percentage of a paper's part, which needs to be removed:

$$\frac{P_2}{P_1} \cdot 100\% = \frac{8-\pi}{16} \cdot 100\% \approx 30\%$$

We can solve this problem in simpler way, for example, we can use exact length value of square's edge, but the percentage remains the same (the percentage doesn't depend on the dimension of paper).

4 Conclusion

During this type of lesson students were highly motivated for work and there was a positive, creative, and working atmosphere. After the lesson, students stop considering mathematics purely as formulas.
Mathematics is the best way to practise thinking and analize problems. After lessons, like lessons with ornaments, students are more ready to analize more tasks than earlier. In general, I always try to teach my students to analize (mathematical) problems in classes, because it is needed for all aspects of life.
When they are thinking of it, they percieve the beauty of its application and

Figure 6: Jewellery inspired by optical ornaments.

optical ornaments inspire them to do tasks more successfully in future.
Solving mathematical exercise is now the same as doing jigsaw puzzle - by using simple elements, you can make more complex structures - in mathematics students apply basic knowledge to solve more complex exercises. Students are actively engaged in the teaching process and successfully achieve set goals if various methods and means of work are combined correctly and creatively. In the mathematics teaching process it is essential to develop the following qualities: determination, motivation, self-reliance, perseverance, and resourcefulness.
After these lessons, students are more focused on tasks and can handle different exercises on their own and not to wait for others to help them. Better working atmosphere is created and better results are achieved on tests.

The pride of all my work with optical ornaments is the jewellery collection inspired by the beauty of optical ornaments (Figure 6). Jewellery like this is always popular, everywhere and everybody loves it, regardless of what the fashion trends are fashionable at that moment. It releases our minds from every environmental impact.
When it comes to my students, they take great pride on the fact that their ornaments, besides my own, can be seen on these pieces of jewellery.

5 Literature

[1]Jablan S, Modularity in art, http://www.mi.sanu.ac.rs/vismath/jablan/d3.htm
[2]Ivić I., Pešikan A., Janković S., Kijevčanin S. (1997), Aktivno učenje, Priručnik za primenu aktivnih metoda nastave/učenja, Institut za psihologiju, Beograd