European Society for Mathematics and the Arts

Newsletter

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Dear Reader,

Many people are fascinated by the unusual and beautiful images arising from fractals. Chuquet numbers («complex» numbers), together with a particular but important class of functions on the set $\mathbb{C}$ *of these numbers, have frequently been used to create these images.*
These functions are called holomorphic. They have a special interest in physics. We shall give examples here of images created with such functions by Tom Banchoff, Géraud Bousquet, Jérémie Brunet, Jos Leys.

As we know, Nature seems to be partly ruled by the principle of stability. An expression of that principle is the fact that physicists consider something called energy, an entelechy, as invariant.

During the first part of the 18th century, mathematicians, in particular Augustin Cauchy, began to deeply study general functions f *of real numbers* x *(real functions) and general functions* g *of Chuquet numbers* z *(complex functions). At that time, people were rather familiar with the main properties of real functions only.*
Given the x*'s and the corresponding values* $y = f(x)$*, one can draw the curve of points* $P(x_P, f(x_P))$*. When the* x *changes,* P *runs on the curve. The speed with which one reaches* P *is called the left derivative of* f *at* P*. The speed with which one leaves* P *is the right derivative at the same point. Given a generic real function, in a standard point* P*, the two speeds have the same value, called the derivative of* f *at* P *: the speed is the same when one leaves* P *or when one arrives to* P*. That is a kind of very local, punctual stability.*
The consideration of functions of $z(= x + iy)$ *instead of functions of* x *introduces a difficulty : one has to take into account simultaneous changes of* x *and* y*. The first idea to overcome that difficulty was to impone the same rule as the one we observed in the pre-*

vious case : whichever be the way we reach or leave $P(z_P, g(z_P))$, the speed remains the same, is invariant. Then the function g is called locally holomorphic. It is holomorphic if it is locally holomorphic everywhere.

In the previous real case, there were only two ways to reach P, since x moves on a line. But now z can move around z_P : there are an infinite number of possibilities to move in a small disk around z_P. In other words, the constraint of stability in that case is considerably heavier than in the real case.

Then one can understand the interest of physicists for holomorphic functions having an energetic meaning.

As a consequence of the basic property of local conservation, more global properties appear as for instance the fact that the integral of g along a path between points A and B, a kind of work, an energy, is independent of the choice of the path.

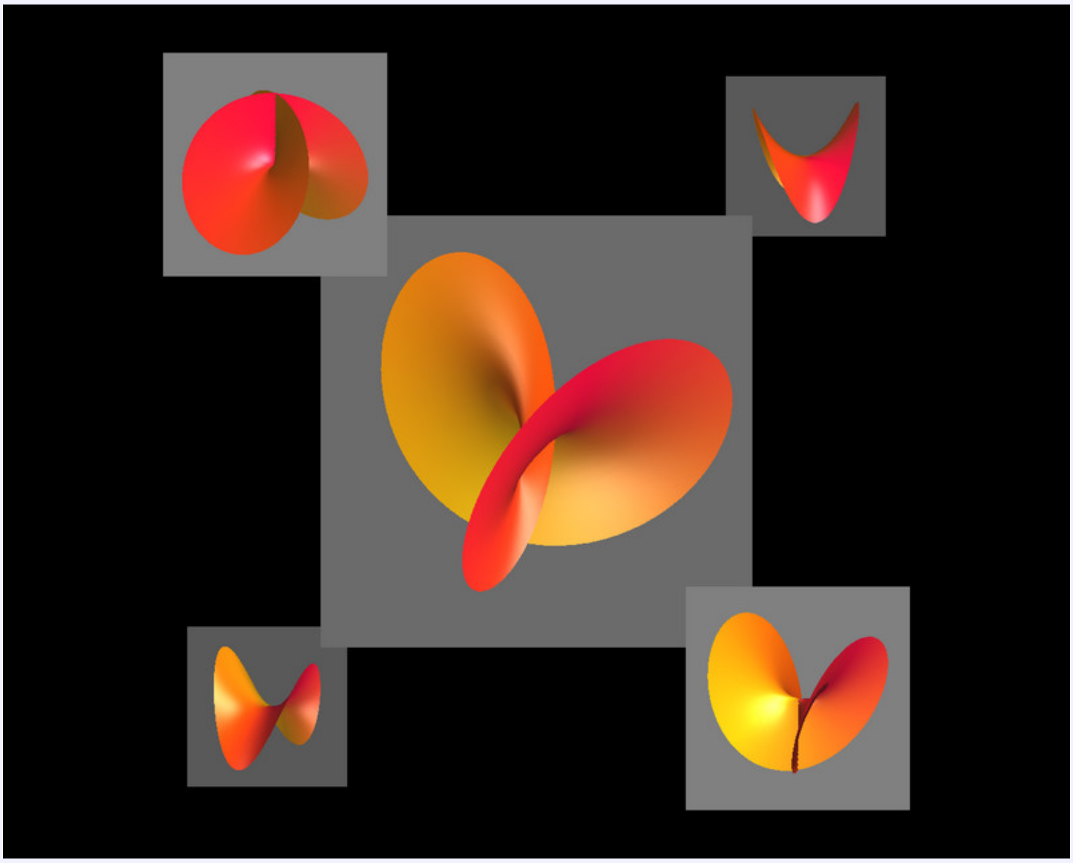
Mathematicians and physicists have been lucky since they frequently use polynomials and quotients of polynomials which are holomorphic. Holomorphic transformations keep local angles invariant. That constraint may sometimes be unrealistic, and non holomorphic transformations have to be considered.

Given a locally non holomorphic function h at z_P one could devide the surrounding of z_P into subdomains U_k on which locally holomorphic functions h_k are sufficiently close to h. The collection of these functions h_k is then an approximation of h, and the derivative of h could be understood as the collection of the derivative of the h_k.

A better way to get the precise derivative of h is to consider h as a function of the two sets of variables : ρ (moduli of z) and θ (angles of rotation), a process that can be applied to functions of any n-dimensional elementary numbers. A very small translation of Philae may have dramatic effects on its stability on the comet Churyumov, a very small rotation of Philae may have dramatic effects not only on its stability on the comet, but also on the possibility of getting some more solar light. These kinds of local moves may also be significant when considering the reciprocal effects of two interacting biological molecules.

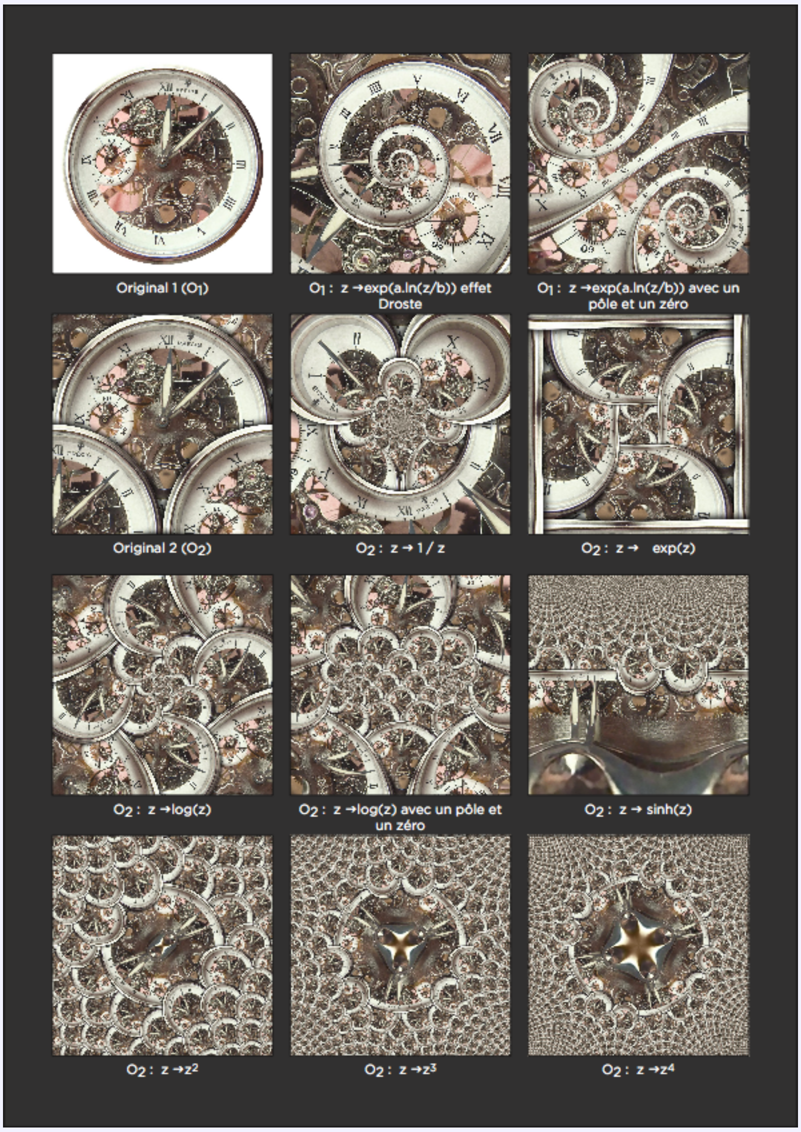
But here, only the rich class of useful holomorphic functions will be taken in consideration.

Best wishes,
Claude



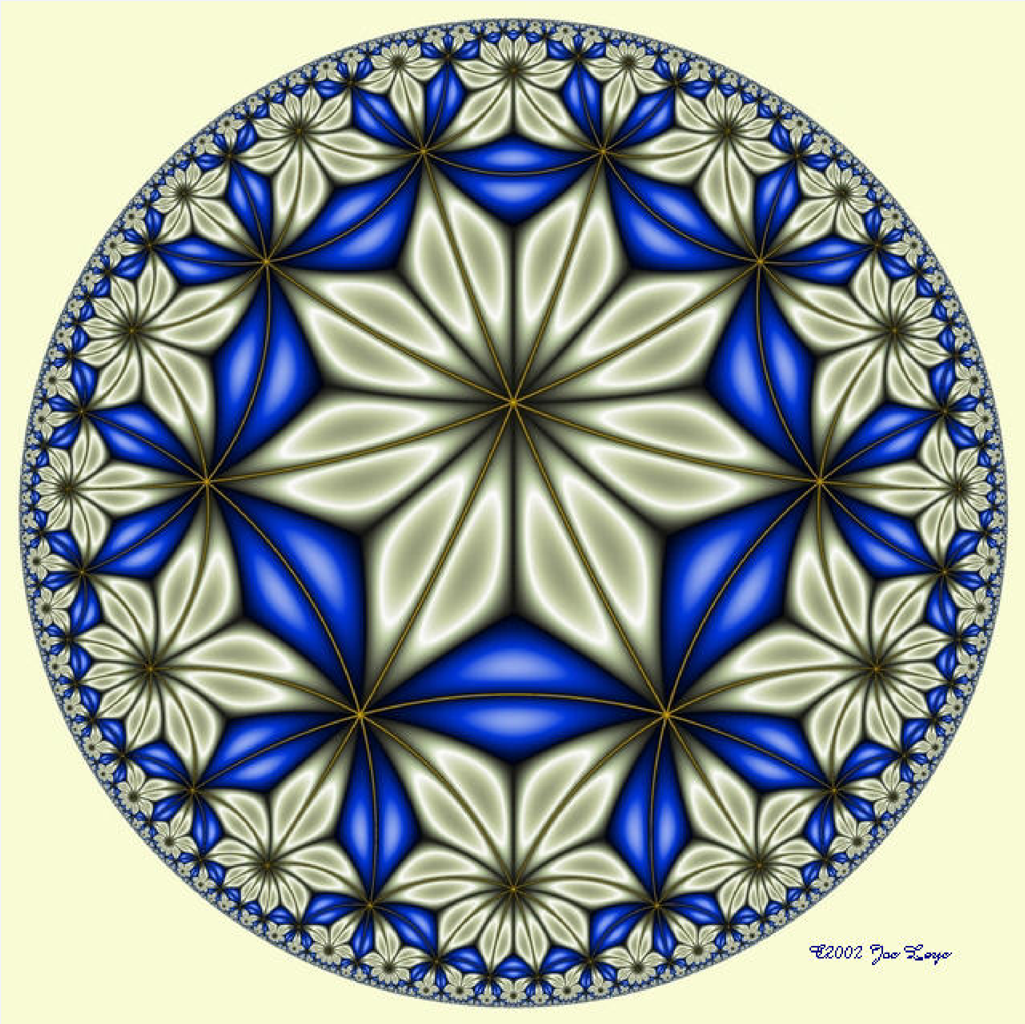
Tom Banchoff : Z-Squared Tetraview

http://www.math.brown.edu/ banchoff/art/PAC-9603/tour/tetra-Z2/tetra-math.html



Géraud Bousquet : Les transformations de l'horloger
http://www.hypatiasoft.fr

Jérémie Brunet : Pythagoras at Alhambra
http://bib993.deviantart.com



Jos Leys : Dîtes-le avec des fleurs
http://www.josleys.com

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