Sur une initiation aux mathématiques par la présentation de quelques concepts fondamentaux incarnés dans les œuvres d'art

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Abstract

This article [1] is divided into two parts. In the first are presented some among the most important concepts and mathematical facts. In the second part, one analyses the contents of presentations characterized by the presence of art works where these concepts are visible, allowing to initiate diverse public, in particular children, to the intelligence of these concepts, thus opening to them the way to a more accentuated and more current penetration of the mathematical world.

1 Introduction

It is established and recognized that image possesses all at the same time, a matchless, immediate, captivating and penetrating power [2]. Then, why not put it to the service of a better if not intimate knowledge, in any case enlarge scope it use to the mathematical world?

What can image reveal here? Situations and objects, known or unknown, to which are associated the maybe unusual properties, sometimes, surprising, in connection with facts and universal behaviors. Revealing then what the image hides enriches the understanding of the world.

In their majority, the courses of study with sometimes old-fashioned contents which are always very limited having no access to this wealth, thus lacking in learning contents.

[1] ≪Mathematics, Art, Pedagogy≫ (in **Aesthetics and Science**, Zoï Kapoula and Marine Vernet Ed., Springer, Berlin, to appear) is a partial summary of this text.

[2] The German philosopher Horst Bredekamp recently published a work, "Theory of the act of image", dedicated to the power of image, more generally of the work of art on the spectator. I would here want to add some remarks. We can consider at first as absent from this text the reference to mathematics. Besides, this power of the image goes very far back in the history of humanity, to when it shared an animistic conception of the world. The image becomes then endowed with a power; it acquires a symbolic status, symbol which one sometimes eventually ends up adoring. Finally the image, the representation in a general way possesses rich semantic contents coming to complete that associated with the often poorer but more incisive, oral speech on certain particular points of the object represented.The synthesis between these two semantics is accomplished at the level of associative areas. The presence and the game of these mechanisms clarify the so important role that image plays in the phenomena of understanding, and makes image an essential educational tool.

If the children and the young people ignore it, it's more likely because of their parents, the general public.

Let us thus use quality image to build and consolidate a communion of warm thought, also to improve the training of the mind by maintaining and by encouraging the vision of objects in the usual space, making known the main mathematical objects with which to better understand the world around us, the reasons why they were conceived.

In the first part of the text which follows is shown in tabular form a list of terms of the most basic mathematical vocabulary. These terms represent objects or a group of important objects by their links in the description and the understanding of the world which surrounds us.

The image allows us to grasp them, to integrate them in us. This list is given of course only for information purposes. Each will draw from it the terms which he will want to derive profit from; will enrich himself according to his own sensibility.

In the second part of the article, are summary and analyses of the contents of various "power points" accompanying oral presentations, in partial answer to the objectives express above.

One can conceive two types of expositions: the first ones centered within the framework of program well conceived intended for the thorough teaching of mathematics, where the mathematical contents of the works of art are clarified and serve as springboard to the deepening of these contents.

One meets elements of such programs in the educational systems which I qualify appropriately as "pragmatic".

In the current French educational system, with theoretical vocation, where the content of programs is established at the national level and applies to all the educational establishments, this first type of exposition is at present hardly possible.

Within the framework of this system, it is possible recently to make expositions of the second type: they are expositions of initiation, having for vocation to widen the listeners' horizon of thinking on reasoned and quality bases where mathematics occupy of course an important place, in connection with other disciplines, in particular those alongside of which mathematics have been established and developed.

The expositions to which this article makes reference are of this second type.

2 Some great concepts of natural philosophy and of mathematics susceptible to illustrations

2.1

The great concepts and the basic mathematical facts are in direct contact with physical reality. They often present an elementary character which makes them easy to reach/understand/access.

One can present them either according to their importance, or according to the epoch when they were released, without maybe, at the beginning, their degree of relevance and implication been appreciated appropriately. These concepts will be marked in bold in the text of this paragraph.

Recent concepts more general and global (categories, sheaves, etc.), take their sense from basic mathematical theories; they hardly lend themselves to illustrations being in direct touch with concrete reality. They will not thus be evoked in this text, an introductory and unpretentious essay.

The fundamental data of the physical world are present in the terminology used by the mathematicians. The most general concepts, which are matters of the natural philosophy, appear to be the ones of **energy**, connected with the available resources, of **change**, and its active principle, the **principle of stability**.

It is a point which will not be approached in this text, connected with the manner under which the organization of Nature appears. The generic term which one can associate with it is that of stratification. Rather for example, to speak about the physical realm, I shall use the terminology of physical stratum. Many terms of the mathematical vocabulary which we are going to meet in the following pages relate in fact to this visual idea, to this general concept of **stratification**.

This is the case for example with the basic terms of tiling, of level curves and level surfaces, of foliation, of subgroup, of singular set.

2.2

The notions of energy, change and stability point to universal data of all the realms, all strata. Before approaching the mathematical vocabulary in connection with energy and change, it is necessary for us to evoke this activity not less universal which is the one of **representation**, by which an object fabricates a kind of ownership of its environment, to assure its better spatiotemporal stability, allowing it to recognize, to escape or to combat what can damage this stability, and to acquire forms of energy necessary for the preservation or the increase of this stability. This representation exists in the inanimate world, certainly in a very primitive manner. For example, the stone which comes to strike the wood leaves a mark on the wood: this mark is a form of memory and representation of the stone.

Representation **Projection** **(parallel, conic, sterographic)** **function, application, morphism** **functor**

Tableau 1

Thus at the center of human activities is the activity of representation. It is in an essential manner a part of the tools developed in the animal kingdom in particular to assure their spatiotemporal stability.

Through the medium of our visual apparatus are created in our brain the images of our environment, mental representations: they condition our behavior according to their beneficial or evil character. Their comparison reveals their common elements. In this way are born the foundations of classifications and mathematics. The relevance of the committed mental processes assures their stability, led to the development of experimentation, theoretical and applied researches through symbolic representations. One finds, natural representations in the creation, the role played by the light first of all, the sound secondly, both characterized by the internal presence of particularly stable periodic behavior. Independently of us, under the influence of the sun rays, the shadows of objects on screens such as the ground/soil constitute these first physical representations. So the action of the parallel rays of the sun on the pyramid plans flattens down this pyramid on the ground: the result of this action, this projection, is the flat shade of the pyramid. This projection is the simplest and the fundamental of representations, the functions or the applications, whatever is the name which one gives to it.

It is constant used by mathematicians who project areas at first on lines (the axes of coordinates), always on spaces of smaller topological dimension, because the examination of the properties of these areas becomes then more easily accessible to them.

The primary geometry is the one of these shadows. The mathematician studies their properties. If one remembers the famous myth of the cave, on the walls of which move the shadows of objects and phenomena, then every man who looks at them attentively is a mathematician.

Mathematics is first a representation by means of drawings of objects and phenomena present in our environment. These drawings are often called *symbols.* The most common of them are numbers such as 3 or 7, letters as f or *ph*i, arrows, the traditional signs such as +, =, etc., the forms of elementary figures such as square or circle.

If A represents a face, the mathematician will show by f (A) its representation made on the canvas by the painter. In certain fields of mathematics, f is electively called a *representation.* Most of the time, this representation appears under the term of *function* or of *application* or *morphism.* Given the usage of representations, their quality of fidelity/conformity is obviously essential: in particular the most fundamental invariants of the source must be present in the image.

Objects and phenomena having many different characteristics, one often meets *faithful representations* only to the one or some of these invariants.

The qualifier used for some of these representations, as for example homéo-, difféo-, iso which follows the term of morphism, evokes at least one of the properties of the represented objects.

2.3

The term ≪*energy*" appears nowhere in the fundamental works of Euclid or in general topology, of arithmetic, algebra, of analysis. It appears in parts of the differential treaties of geometry stemming from mechanics, and in very great majority of works called applied mathematics. If it is not visible in the fundamental treaties, it is nevertheless present at the background in a potential manner. The concept of energy is present in arithmetic which studies the properties of classical numbers, and first of all, those of integers. An integer is primarily an indicator of a certain presence. Not only it constitutes a primary representation of the presence of a certain shape of materialized energy, but, in its cardinal aspect, it also gives a first tangible evaluation of that energy. Associated with a number, the square of a length is nothing other than the work of a force along an appropriate pathway (cf [1])). It is thus one of the forms of expression of a local energy. This elementary fact reflects the presence and the underlying role of the notion of energy in the construction and in the implementation of the basic mathematics, in particular the geometry. The concept of energy is also and implicitly present for example every time one speaks about space, and thus in topology and in geometry, because energy is embodied in a place, in a spatial domain, and that this place is not physically conceivable without the presence of energy.

Topology deals primarily with the container of energy, independently of the way the energy structures, inserts it with forms locally or more global.It is called general topology. The consideration of the precise manner of energy allows specifying the manners by which it is embodied in the shape of objects of which it models the outlines and the internal structure. One deals then with geometry, and also with topology when one is interested in the specific but intrinsic properties of forms.

In the second table, reflecting partially on their appearance during the cause of history, are presented the primary concepts at first connected with the notion of energy and relating to objects. The inventory and the study of their forms follow, leaving little by little the primary appearance to penetrate deeper into their internal structure to show their more intrinsic properties.

Objects considered here present characters of rigidity: their spatiotemporal stability confers on them the possibility of being analyzed and patiently observed for a long time and registered in our memories. This stability is at the origin of Aristotle's maxim: "the inanimate precedes the animate". In fact, the description of the inanimate, which leaves itself to be contemplated, observed, assimilated, interpreted will be primarily unlike the description of the animate, more imperceptible to our senses, sometimes even short-lived. The cinema was born after the photography, Fermat and Huygens lived almost two thousand years after Euclid.

2.4

The consideration, approach and the control of change and of movement gradually became accomplished in the course of this last millennium, with an increasing rhythm. The physicists did not stop finding new processes for accessing the observation of faster and faster phenomena.

In the same way, the mathematicians produced more and more works dedicated to the study of motions. The term "change" does not appear in the technical vocabulary of the mathematicians.

This vocabulary relates to most of the modalities of this change: change in the position, spatial, temporal, spatiotemporal change in only the form, change in the data which define the positions and the forms, and on the contrary, the absence of change concerning such or such characteristic property of the object or the phenomenon examined.

This vocabulary thus concerns the *movements* suffered or one which is enforced: in space and in time, they bear rather the name of transformations; when it is about the shape and the parameters which define it, one speaks rather of *transformations*.

Global evaluation of energy attached to these objects: **number**
sum, power, polynomial
quotient, fraction
integers, rationals, transcendents, reals, etc…

associated theories: **arithmetic, algebra**

Local evaluation…………………………………………..: **lenght**
metric, vector, tensor

Apparent shapes with metrical data: **dimension**
polygones, polytopes, cones, spheres, disks, tori, Möbius, Boy,
curves, geodesics, surfaces, genus, tilings, knots
curvature, torsion, level curves and surfaces
vertex, singularity, singular set

associated theory: **geometry**
(the different types of geometry)

Apparent shapes without metric data: **neighbourhood**
open space, closed, interior, boundary
fiber space, leaf, foliation
handle

associated theory: **topology**

Generation, technical construction of objects: **basis, generation**
extension, fiber
unfolding
inflation, pinching, attachment, identification, thickening, covering

Tableau 2

Moves **transformations** **translations, rotations** **expansions, homothetic transformations** **deformations**

Tableau 3

It would be by no means out of place to put also in this table the techniques of the construction of objects previously encountered within table 2 and which come along with diverse changes. As for the techniques of study of the spatiotemporal movements, they belong to the *theories of the differential equations* and to *partial derivatives equations* . In the first of these theories, one establishes in every point in space the speed of change according to position only.

In the second instances of these theories, one establishes the instantaneous value of significant parameters according to the variations of the value given for other parameters not less significant.

All these theories, by their local character, are principally the subject matter of *analysis*, even though geometry and algebra often have their important part in their study.

Analysis can be defined as the fine study of representations of objects and their transformations by number. It decomposes the object into smaller and smaller elements, and then adds these elements to reconstitute the whole. As these elements, these fractions of the whole are of the same nature, the numerical representation of each of them conserves the properties of similarity.

Thus one carried out what is called a *fractal decomposition* of this whole. It remains to make sure that the sum of the so defined elements, called by a little bit unfit term *series* when this sum consists of an infinite number of elements, corresponds well to the whole: it is the problem of the serial *convergence*.

To return to change and movement, to their description, which, from the intrinsic point of view, belongs to the theory of *dynamical systems*, let us retain at the moment only these three terms:

Trajectory **attractor** **basin of attraction**

Tableau 4

The first of these terms is related to the term already encountered *leaf*, trajectories can be collected together as in leaves which go through space in which there are movements. The happy choice of this terminology doubtless comes from physics, from electromagnetism, a magnetic leaf being a magnetized thin blade. The second of these terms, attractor, is often qualified as *singularity* or as *singular set*.

2.5

If the energy relates to the resources necessary for the deployment of any form of activity, the principle of stability is largely situated at the origin of this deployment.

One shall find in [2] a record of these two notions. They are present in all technical and science fields and even today in daily life: they are endowed in a way with the gift of omnipresence.

Therefore, there is no precise and general definition of these terms corresponding to their usage in mathematics. Often, such or such study makes reference to these notions by introducing and by naming for example "energy" a particular mathematical entity adapted for this study.

If the notion of energy is difficult to apprehend - energy is an entelechy, stability is associated with a principle and with organizing activities which seem universal. Objects are at first recognized by their appearance, their external shape. One proceeds further to the examination of their internal constitution. But an object cannot exist without showing certain form of stability. In the natural world, the concept of stability appears in the form of an essential principle which goes back to Plato, which only Spinoza reused, and which one can formulate today as: "All object struggles to preserve itself in space and through time" This concept can be deployed in many manners.

The first embodiment of the stability is in the form of the invariance [4] which will be opposed by the non-stability, the non-invariance.

The primitive and visible shape of invariance displays by the static character of certain objects, by the fact that they are inanimate, by their rigidity, by that of a certain number of their properties. It concerns for example the mass for an object of the physical material world, about the Self for an object of the animate world. Let us take all the triangles of the Euclidean plane: it is a family of objects, seen as globally as an object, here rigid. The fact that the heights, the bisectors, etc., of every element of this family are convergent is an invariant property of this family, of this global object. The spatiotemporal permanence of objects and phenomena shows itself by that of their internal properties, more or less revealed by the appearance of these objects and phenomena, their external morphology. This appearance is more or less immediate; we were able little by little to create diverse tools allowing us to see according to the degree of smallness in size or remoteness of the objects and phenomena. The first appearance is that of the form. As shown to us by the observation of the natural representations which are the shadows, one of the primary characteristics of the form is of proportions (or still relations) existing between its diverse elements. Thus one of the primary qualities of a representation will be the respect for these proportions. The statement attributed to Thales expresses the fact that the proportion between two elements of the original form is equal to that between their respective representations. All these are accomplished in an instantaneous manner and are thus maintained in a static state or in a state of uniform velocity. To this, it will be necessary to add the influence or still the modifications of the conditions of observation on the establishment of proportions. So, for example, always in a static state, looking at the elements of an object in two different but fixed directions, leads to bringing in, through the constancy of what one names the birapport, the invariant proportion between these two directions of observation. Thales and Pythagoras are practically contemporaries, and it was in their times that the mathematical notion of proportion was set up as universal concept, that the rudiments of a theory of proportions were developed. With Archimedes, approximately three and a half centuries later, appeared, in touch with the proportion, the primary elements of the theory of equilibrium of forces, equilibrium which guarantees stability. The Greek term of $\sigma\upsilon\mu\mu\epsilon\tau\rho o\varsigma$ suits perfectly the description of this situation.

In a general way, the observation of symmetries in appearance and structure reveals the presence of internal forces in sufficient harmony to allow a certain spatiotemporal invariance. From there comes the frequent and happy use in physics of the theories based on the notion of symmetry. It is a primitive symmetry, the one which corresponds to the presence of only two opposing forces. It corresponds to a form of dualities in nature, and leads to envisage for all objects, at least potentially, the existence of a kind of double, of twin, but endowed with opposite orientation or at least different.

In the physical world, the annihilation between a first subject and the second belonging to the dual family happens as soon as they meet if their supports are different. In this situation of perfect equilibrium between two actors, it is indifferent to support one side of view or the point of view of the other. A permutation between both actors, corresponding to a perfect symmetry between these two actors, leaves invariant the global phenomenon. This is the reason why one assimilates permutation with symmetry.

The generalization of this type of situation of perfect equilibrium between n actors leads at first to the creation of set of movements that are permutations between the n elements of any set.

The codification of three of the elementary properties of these permutations (a permutation of a principle, said neutral, which leaves invariant all the elements, the fact that to any given permutation corresponds a symmetric permutation which returns initial elements to their situation of departure, the fact that one can compose between them permutations) leads to the general definition of a group, a set of transformations which satisfies the three properties explained above.

The invariance by effective symmetries with regard to the "axes of symmetry" of dimension 1 (as the straight line) or superior, by diverse movements as translations, rotations, along paths which return to their starting point, the invariances which preserve the energies as the lengths called also metrics, as "quantity of movements", are the main constituents of the groups.

Representations, to be a little bit faithful, must respect the properties of stability of objects sources. One is therefore not surprised, in their formulation and for many of them, by the frequent reference to the groups or the subgroups of symmetries attached to these objects and by their presence (as in automorphic and modular functions). As we have already noted, the periodicities of appearance of elements within an object, during a phenomenon underlines the presence of internal invariance, of stable cycles of repetition of mechanisms, characterized thus by groups with cyclic structure.

The appearance brought not only to evidence the presence of more or less perfected symmetries and indicted, associated, at first, with the stability of the object seen in its totality.

It also attracts attention to the opposite of global nature, on the local, the most local which is possible called singularity, as already discussed. The singularity [3] is an element among the most important morphology rigid or inanimate: it is isolated at first - all the singularities form a set of zero measure. Characterized by properties of ≪extremality” ([7], §2.4), it is visible and attracts attention. It is also from singularities that are born and unfolded the forms and the movements: they play a role of an organizing center both on structural and functional plans, associated with the fact that the potentialities of local transformation are particularly high, unlike what happens at regular points. The singularities are thus also associated with phenomena of bifurcation [5] [6] where structures and completely new way appear.

For example, the values of the elements of a group of symmetries can depend on parameters evolving with time. It could happen that, for some of these values, symmetries disappear: these values have the status of singularity. These values of parameters can form important sets with certain parties of which are associated with particular types of forms and behaviors. When we reach the boundaries, the limits of these parties, then appear in a more or less immediate way changes of morphologies and more or less deep behaviors. One sometimes indicates by the name of *set of bifurcation* or *set of catastrophe* the set of values of the parameters in which happen these important changes.

When important sets of parameters have the group structure, these significant parts also have generally a group structure, and form subgroups of the global group. They bear then the name of *Galois subgroup* of the global group.

The phenomena of evolution characterized by the growth of certain structures, of certain morphologies, of certain values, appear in mathematics under various vocabularies, such as *unfolding*, of *adjunction*, or *extension* ([7], §2.6), in particular for this last term in the theory of numbers and in logic.

Such phenomena are even sometimes modified thwarted by the arrival and the presence, in certain stages of evolution, of particular structures with qualities of obstructions, of values of parameters inducing breaks of symmetry and of appearences, of bifurcations of behavior. They correspond to the sets of bifurcations evoked above.

The following table summarizes the essential notions associated with the principle of stability:

	Principle of spatio-temporal stability	
associated with	**symmetry**	
singularity, singular set		**group**
obstruction, bifurcation		**cycle, period**
bifurcation set, set of catastrophies		**Galois group**

Tableau 5

2.6

Let us end this section with considerations which concern certain general properties of the mathematical objects. They have happy consequences, in particular methodological consequences, on the way of handling problems.

There are a priori several manners to represent the same object, in particular according to the tool of representation used. In every manner of representation is associated an original look and the putting into value of the discovery of a particular property, maybe unobservable everywhere by any other mode of representation.

duality
analogy
approximation
extremality

Tableau 6

There are a priori several manners to represent the same object, in particular according to the tool of representation used. In every manner of representation is associated an original look and the putting into value of the discovery of a particular property, maybe unobservable everywhere by any other mode of representation.

An observer looks at you, as well as your image in a mirror: the observation in the mirror will complete the information which he has of you by the direct observation of your person. The physicist-mathematician may say that your image and your self are in *duality*. Modify the properties of the mirror; you obtain an infinity of possible dualities. But it is clear that certain modifications will be less useful and relevant than others.

Let us now take a rolling hoop on a plane surface. The vertical plan in which is the circle cuts the horizontal plan on which it runs according to a straight line. This line has only one common point with the circle, that of where the circle touches the plan on which it runs. In other words, every point of the circle corresponds to a particular straight line called its tangent at this point. We can thus describe the circle by all its points or by all its tangents. The correspondence between all the points of the circle and all lines tangents is one-to-one: at any point of the circle corresponds one straight line, and at any point of the straight line corresponds a single point of the circle.

When such a bijection between representations exists, one says they are *dual* to each other.

In this case, in every point c of the circle is associated a straight line, a visual geometrical element, but also, in its numeric representation, a linear function $f_c(x) = a_c x + b_c$ which describes the straight line passing through the point c.

Let us note that in the highest point of the circle, characterized by particular values of angle or the coordinates of this point, corresponds also a particular linear function.

This correspondence between space of functions and forms, their local properties is widely used because it allows transposing a priori properties from a representation to the other. Demonstrated in one of the system of representation, one is then assured of their existence in another adapted system of representation. The mathematicians say sometimes with good reason that they proceed by *analogy*.

The physical world, infinitely big or small, remains to us inaccessible. Already nobody knows what an electron really is, and then to know what takes place at the level of Planck lengths of the order of 10-35 cm is not less mysterious.

[3]

By making the hypothesis of stability of the constitutions of elements as they become smaller, the mathematicians established undoubtedly rough representations of most of the objects and phenomena. The formal process preserves its operational character as long as one does not go too far from the approximation. Most of the objects which we consider present characters of finiteness. For a given direction of observation, this finiteness is translated by the measure (relevé) of extreme positions and values which are impossible to exceed. These properties of extremality confined on objects the characteristic of singularity. The systematic technique which consists in clearing extreme elements and situations is always rich in instruction.

2.7

One can regret the absence in these tables of very numerous terms associated with deepening of the notions presented and which are of major interest. But it could happen that one of these terms is evoked in a pertinent way during one of these exposés.

One could thus occasionally, for example, say a word on homotopy and groups of homotopies, or to speak about the inversion and about its link with symmetry, or even more simply to speak about perspective. It is necessary to recognize that terminologies and intermediate facts linked for example to number theory, to the most developed algebra, lend themselves badly to the esthetic illustration, while, nevertheless, the obtaining of the solutions of polynomial equations offers an endless field of brilliant forms so unexpected but fascinating.

3 Some comments on the content of the two lectures

These exposés, in principle of one hour maximum duration were made before young pupils, of ages from six-years to fifteen years. The adults present were equally interested. Couldn't we then rightly deduct that these exposés could interest any public? The associated Power Points allow reaching out to the essential points of these presentations. But of course, it sometimes happened that, consideration to the public, its reactions, consideration from the inspiration of the moment, other considerations, in particular of general and "philosophic" order, were fittingly inserted into the presentations.

Here is a brief evocation of their content through the keywords appearing in the text of the Power-Points. This reading of a kind of Prevert dictionary will

[3]Robitaille P.-M., Crothers S. J., *TheTheoryofHeatRadiation*" Revisited: A Commentary on the Validity of Kirchoff's Law of Thermal Emission and Max Planck's Claim of Universality,Progress in Physics, v. 11, p.120-132, (2015), *http* : //*www.ptep* − *online.com*/*index_files*/2015/*PP* − 41 − 04.*PDF*, the "constants" of Planck have nothing universal. Besides put rest the use of the integral calculus in physics of the extremely small...

doubtless embarrass even worry many. Maybe, after having knowledge of the end of the text of this section, they will feel a little better reassured.

" Happy New Year/Good year Bonne année ": balls, spheres, cones, circle, stereographic projection, knot, deformation, homotopie, singularity, épicycloïde, cube, dimension, n-cube, face, hyper cube and evaluation of the number of its faces, trefoil, projection, weight of a projection, orientation, dextrorotatory, levogyre, Möbius strip, cylinder, base, fiber, fibred space, breather, Klein bottle, tiling, hyperbolic disk, Boy surface.

Important Affirmation: scarcity of singular points.

" Mathematical Pastry /Pâtisserie Mathématique": cream slice, parallelepiped, cube, prism, sphere, singular point, ball, dimension, curve, Thales' proportion, cylinder, tangent cone, Conforming transformation, torus, knot, Trèfle's knots, thickened sheet or plate, vector, displacement, vector field, Group structure, Baker's transformation, the pavement, the auto-similarity, the hyperbole(hyperbola), the regular polygon, the brioche of Boy.

Important Affirmation: possibility of going through areas, stereographic projection with elements of demonstration, Thales theorem (recent dynamic version), there are 17 crystallographic tesselations of the plane, the plane can be tiled by any quadrangle (with proof).

These exposés present common characteristics.

3.0 They are not traditional courses the contents of which must be learnt, what is learnt by these contents is not subjected to check by interrogation and examination.

3.1 The first feature of these presentations is their cheerful character: brilliant colors, humor, relaxed conversation, of the unexpected starting from the title, one puts the audience at ease and one is confident. See these first three images of the presentation "Mathematical Pastry/Pâtisserie Mathématique" which is addressed to the teenagers:

or these first four slides of "Happy New Year/Good year /Bonne année ", conceived for a younger public:

The tone is given from the first contact with the public.

3.2 Another of the general characteristics of these presentations comes because they are presented to finally a public ignorant of what mathematic is.

Therefore the need for little reminders obviously so simple and short as possible on what is mathematics, what the mathematicians do so as to dissipate fears, throw out apprehensions, false definitions which cause rejections.

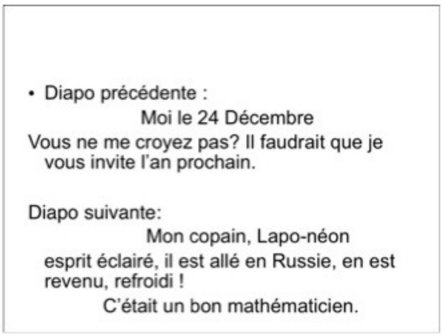
The contents of these lines are often widely enriched during the exposés by affirmations of elementary considerations of how the mathematical universe is deployed.

What is Mathematic? Remember this: simply a little bit elementary manner, but effective way, to describe and to represent the essential features of the world which surrounds us.
It facilitates the understanding and the forecast.

Mathematicians demonstrate it, that is, know how to explain why it is so. A statement which the mathematicians demonstrate is called a theorem.

Mathematicians look for the universality of their conceptions and their affirmations. They generalize.

There are two types of geometers: the complete, and the incomplete or intrinsic or*topologists*. The complete geometers take into account, not only the shape, but also the metric data of objects, the distances from one point to



another. The topologists are not a priori interested in distances. They study more intrinsic, more fundamental properties. They look for invariant properties by deformation.

For instance, an essential property of the circle, for the mathematician who is hardly interested in the questions of length, distance, is the following: if I walk on the circle, leaving by any point of the circle, I return to my starting point. Such a property is independent of the radius of the circle, of any deformation of the circle which does not break it into disjoints pieces.

3.3 The third characteristic of these presentations is to make known to the public, diverse objects among the most basic which the mathematicians manipulate, in particular by showing how their presence is familiar around us.

For example, in "Happy New Year/Good year Bonne année", we show funny characters, buildings and famous paintings as well as different mathematical objects, and then one incites the listeners, by guiding them, to reconstruct the common objects with the help of the mathematical objects highlighted.

The corresponding mathematical forms acquire then a physical reality: they are immediately imprinted in the listeners' mind.

The new vocabulary, the meaning of its terms is immediately assimilated.

One also used the same method in Mathematical Pastry/ Pâtisserie Mathématique, appealing to additional activity of our natural appetite.

3.4 We deepen a little the knowledge of these objects. It is a question primarily of showing the presence of the universal properties which objects share, essential properties of which one is not conscious at first sight, as the presence within all these objects of singularities.

The elements of text concerning these singularities and which appear on the slides are very concise; they do not take into account all elements of information given to the listeners about these singularities.

At least, one sees emerging the general affirmation of their scarcity.

The use here of commented animations, for example, stimulates the attention and allows making the educational message to penetrate better.

http://www.josleys.com/gfx/Danse_03.mov,

Even there, comments, made orally, concerning the properties of movements do not appear in texts, showing in particular by the combined presence of translations and rotations and illustration to articulate the general theorem of Aristotle - Liouville to the public.

The use of the images allows to introduce and to quickly show the significant properties of the representations of mathematical objects, in particular by projection: the parallel projection which leads to the affirmation of the theorem of Thales on the preservation of the metric relationships of lengths, the stereographic projection which accompanies the conservation of angular distances

3.5 The latter appears in both exposition, but the second, "Mathematical Pastry/ Pâtisserie Mathématique ", is of a higher level than the "Happy New Year/Good year Bonne année ": proofs or their sketches are present.

But we do not intend to introduce them abruptly, on the contrary one tries to show their origins, the motives. As regards for example the stereographic projection, one makes the public to participate in the progress of a very hypothetical approach of the creator of this projection, who would have paid its attention to the shade of a sphere lit by the sun situated on the north-south axis of the sphere, approaching the source of the conical projection until placing it in the North Pole of the sphere.

By this example, one intends to show, not only, the approach of the thought which leads to the discovery and to the shaping of an affirmation, but also and primarily the physical environment which allowed, aroused the implementation of this approach.

In another text, a tale entitled "The garden of delight"

(http://www.math-art.eu/Documents/pdfs/pythagore.pdf)

one also describes a continuation of steps of thought leading to the statement of the Pythagoras' theorem, this time connected to a scientific environment and to a social environment situated in their epoch. In other words, this other feature characterizes the exposés: they try hard when the possibilities are present, to revive the historic development of mathematics in its most diverse aspects.

3.6 The exposés also offer the possibility of understanding the mathematics, by the universal ideas which they carry, as a tool of comprehensibility which goes beyond the simple physical world from where they arise, but also as a source of modesty and caution.

An example is given by the first comments which accompany the animation:

http://www.josleys.com/gfx/Danse_03.mov.

The lesson to be learned of this phenomenon:

- · Never take the Shadow, the Appearance for the Reality, the Number for the Fact it

I would want to say briefly here how much any training of thought, by mathematics among others, by daily habit of thought and by the preconception which it could eventually impose, could limit our understanding of the world.

3.7 The contents of the expositions are rather rich; in fact one hour is not enough by any means to go through and to comment the contents of the Power-Points. It happens notions are presented which do not appear on the tables given in the previous section. This diversity, even if it could distract the least attentive, allows for making a better glimpse of the wealth of the field of mathematics.

If the theme of foliation is approached only during the exposition dedicated to the pastry, that of the plane tilings is present in both exposés.

The physical motivation in the study of these tilings is not figuring on the paper of the Power -Points but is given orally: study the way Nature fills the space. Crystallography brought a first answer to this question, and crystallography played a driving role in the development of the theory of the groups. Curiously, one makes apparently almost no mention, in the works on tilings, intended for children or not, the connection between tilings and the original physical problem: a plane tiling decomposes into layers of plane "crystals". Such a layer is the equivalent of a leaf thickened by a foliation. Such a thickened leaf is the equivalent of one smooth layer of crystal.

The subject of the plane or not motives (polygons, polytopes) which constitute tilings, the topic of tiling offer the unlimited possibility of conceiving images of great beauty, and also to reach the mathematical technical tools which allow their construction, and primarily the notion of symmetry.

Diverse animations show it different possible uses: that of Jos Leys which allies symmetry, topology and music :

http://www.josleys.com/Canon/BachCanonL_final.mov

for the great pleasure of all the listeners. For these excellent reasons, doubtless there is hardly any school program which today does not mention tilings and their motifs.

3.8 The presence of animations is another property which characterizes the exposé ≪Happy New Year/Good year - Bonne année" . The animations create at the same time soothing moments of break in rhythm of the presentation but also contribute to fix attention.

One will observe these animations are of interest by putting to use the synergy of several fields, to show in an almost simultaneous manner properties or facts apparently from different specializations.

With regard to situations where one hangs only on a single property, the enrichment is possible on the condition of lingering over the animation, reusing it to show all the facet, details, and analyzing the relationships between them. Generally speaking, the animations are in too insufficient numbers.

Two reasons can explain this fact: on one hand, to conceive an animation which meets certain objectives is sometimes impossible, and on the other hand to realize it demands some know-how and uncommon qualities.

In this particular case, it was an opportunity to be in relation and in collaboration with Jos Leys whose fame needs no more further demonstration.

It is naturally the evocation of an animation which will conclude this last paragraph.

http://www.josleys.com/gfx/Tore_CB_01.mov

It has the advantage of showing us a mathematical object, a torus skillfully coloured, coloured garnet, a foliation which is associated with its singularities, famous architectures, the impressive Roman arenas of Nimes and Verona which tourists from the whole world do not miss to admire, a very important mathematical notion, the deformation.

Deformation in the vicinity of singularity leads to bifurcations, to the creation of new morphologies.

3.9 The last characteristic feature of these expositions is obviously the presence of very beautiful images, bright, twinkling when they appear on computer screens. We are in hundred places of learning ex cathedra where one remains motionless in the face of paintings which were black, today in the softer tint.

In the hands of talented artists, the mathematical objects are embodied, reveal their surprising forms through lighting effects, and as one discovers it for example in the compositions of the Cubist painters, the same mathematical objects skillfully placed and combined with themselves become expressive constituents of fascinating representations completely charged with humanity.

These exposés are thus of another nature to those which one meets in the standard university institutions. It is doubtless necessary to compare them with similar shows for example to those of the circus, the magic shows where the strangeness gets married to the wealth of light, leading to creation of a kind of atmosphere as poetic as the one which impregnated all the work of Chagall.

Shows one must see again, not innocent shows, shows which arouse the reflection, which, with their procession of new notions, can leave indelible marks in the memory, another educational advantage.

The author is the only one present in this bibliography. The reader, having acquainted with the quoted texts, will maybe indulgent towards him.

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