THE PEDAGOGICAL VIRTUES OF MATH AND ART EXHIBITIONS

Claude P. Bruter

Abstract

This article deals with exhibitions of artistic mathematical works as useful tools to fight against the disaffection of the public towards mathematics.

1 Problems of today and of yesterday

The subject which we will treat in this article concerns disaffection of various publics towards the sciences, and more particularly towards mathematics. How to succeed in reconciling these publics with mathematics in particular, the role of which is fundamental in the understanding of our universe, of our environment, in the conception, the creation and the implementation of all these means among others technical which facilitate our lives and which prepare our future? What are the contents of the messages which we should send, be assimilated by these publics, and how can we achieve this?

Many of us share the conviction that artistic work can be a useful tool for the diffusion of mathematics, their initiation, their teaching. But it is a fact that many of our fellow-citizens, of our decision makers, even some of our colleagues, are not convinced of the merits of that approach, either due to lack of judgment and reflection, or quite simply from ignorance.

This article intends to address those not very familiar with mathematics and arts decision makers, and those colleagues who are not totally convinced. Being mainly focused on exhibitions, it is a very partial introduction to the subject.

2 The foundations of the argumentation

It is obviously impossible to handle in detail and in a short article the multiple aspects of the elementary questions which have just been introduced. I shall have to content myself with making here one excessively short and synthetic presentation of some points of view and partial solutions that I have already had the opportunity to expose and to experiment with.

The following arguments constitute their common foundation:

an argument drawn from general biology: in all the aspects of development and evolution, ontogeny recapitulates more or less, and is a reflection of phylogenesis, in particular its important accidents. As a consequence, in order to teach and to lead to a better understanding of the most modern theories, it is necessary to carry out an intellectual journey analogous to that which has accompanied, over time, the progressive implementation of the basic notions and the essential facts, since their most distant origins.

an argument related to our physiology: connected to the general vitality of the body and its driving system, the primary sensitivity is that of the senses, enabling us to immediately comprehend our world, its dangers and its advantages. Very young children, up to four-five years of age, have a global vision of space, a spontaneous prehension of 3D. The view fixed to the plane, the mental exercise, practiced during school years, the young years of maturation of the brain, and restricted to what takes place in two dimensional space, tend to make the imaginative vision in the three dimensional space difficult later.

an argument also related to our affectivity: the most of us assign to the objects that we meet aesthetic qualities to which we are often sensitive to varying degrees. Our strongest and most long-lasting impressions are generally those which have struck our emotions.

These primary concepts are strongly present in the first part of a work entitled *Comprendre les Mathématiques* [1], about which the late Gustave Choquet, announcing his "admiration for this beautiful success", wrote, this "book merits to be read and reread".

These are also the underlying concepts of the ARPAM project, in the form of exhibitions of works, conceived from mainly mathematical considerations, remarkable enough to be able to be qualified as works of art.

The value of a work, whatever it is, is measured by the quality of its realization, the originality of its conception and by its contents. It is the unexpected which strikes and which attracts, here is for the appearance, but in the end, we consider its significance and its contribution to universal well-being, knowledge and intelligence.

It is not always immediately obvious to appreciate this value, to recognize it and to comprehend it in all its aspects. The guidance of a third person is never useless to reveal these contents.

3 Some critical observations

Do we flee from the mathematics? Why? Most likely because what we teach is hardly apparent in the physical world and is not physically perceived, has no immediate meaning for the body, does not awaken the senses, requires an effort for the spirit to give it any tangible reality. The effort is mostly without profit, disappointing, disheartening for some. What we show is fragmented, without reference to history which shows the justified genesis of things, thereby confirm their presence. We do not understand where we are being directed, or how this present joins in an organized ensemble, reassuringly solid.

Its embodiment in the material object gives to the mathematical abstracted object a physical reality, the image and functional character of which can be all the more comprehended and felt by the spectator. If this functional character is absent, only particular properties of this material object will allow its image to join the memory, to assimilate to the cleanly vegetative sense of the term the essential features.

It will be the case in particular if it presents asserted aesthetic characters.

These are thus the principal reasons for which the mathematical objects will be accepted, will have their recognized presence whether it is in the form of models and small sculptures or representative drawings emphasized by the genius of the artists. We shall no longer, we shall wonder.

In the light of these data and of these common sense facts, it should come as no surprise that the young generations show a lack of interest in mathematics. The current trend, under the pressure of some professional mathematicians, physicists and engineers, is to favor calculus to the detriment of geometry, while to the young children, the number is almost meaningless, unlike that of the figure which has a pregnant physical and affective meaning. In the current teachings of many countries, the number, the letter and their use take it on any other consideration. In the mind of a child what do these two attached symbols 11, and the number 11 mean: a friend, the cat, a car, a bar (of chocolate)? And $x^2+b=0$ or $\sqrt{2}$: that is drinkable, that is edible, we can throw it then catch it? It is through the exercise, mental calculation, the acquisition of the multiplication tables that the child can become familiar with number, even if the numbers have no significance for him. On the other hand, the drawings of a triangle, a polygon are physical structures which make sense, to be associated with the shape of common objects.

Thus rather than to eliminate it, it is on the contrary the "monstration" of shape and its properties that has to be the object first and foremost of all the attentions in the early education of mathematics. The simple drawing of a circle is enough to immediately show one of its beautiful properties, the most wonderful doubtless, the one that surprisingly the circle, the only one among the infinity of flat shapes, has the privilege to possess. The role of the mediator here will be to help the creator of the circle, the attentive spectator, to make the formulation of its observation mature and to help him to express even this formulation in exact terms, to give birth would have said Plato to what is being developing in his mind.

4 The charms of the exhibitions

These exhibits allow the public first to discover the richness of the mathematical world, and second to allow that public to get acquainted with some quite modern mathematics, completely different from what they learned at school, and without any mental stress. The exhibits are thus by no means boring, they are felt beautiful and amazing, intriguing, enriching, but of course they may leave a feeling of dissatisfaction from the fact that the meaning of many of the works is not quite understood, the feeling that a large gap remains between the visitor and the content of the works he has been admiring. If the psychological position of most of the visitors towards mathematics remains somehow ambiguous, in any case, the psychological resistance against mathematics has diminished, and this is a true first success.

The exhibitions of models, small sculptures and printed works, whatever the medium may be, show mathematical objects of generally recent conception and discovery, emphasized by illustrators and quality artists.

These are main advantages in their favor. They present a wealth and beauty that are often missing in the oldest objects, and it is their very novelty which attracts the curious and the crowds. Has not the term "fractal" become a household word for example?

That they were the object of recent attentions on behalf of the mathematicians, make us think that it is the most current mathematical theories which were of use to their discovery and to their study. In other words that in the presence of these objects, we are in fact at the heart of modernity. And if thus we bring to the public some at the same time simple but penetrating explanations on their subject, the same auditors will be pleased to feel in sync with the most astute current events. Why then would they reject mathematics? They are beautiful, multi-form, and thought well in their accessible foundations.

Models and small sculptures have the advantage over the printed works for they can be touched, manipulated and examined from every angle. Some of them can be knocked down and built up again as one pleases, adding to their charm for the handymen. They then become occasional pastimes.

The most interesting of these objects are doubtless the ones which are realized with threads. We can enter inside objects, allowing us to view hidden aspects in their structure.

And among these objects in thread, are the deformable ones which illustrate additional properties. They can take unexpected forms, surprising dynamic

behavior. Children in particular love to manipulate these objects, which have now an appreciated playful side.

Finally, rigid or not, suitably lit, their shadows on well chosen surfaces add to their charm and to their interest, so much rich is the mathematics of the visible outlines.

I do not doubt, in this place of my presentation, that auditors and readers are perfectly aware of the value of the exhibitions to contribute in an effective way to easing the fears felt by a lot of public towards mathematics, and to lead to a positive vision of our scientific universe, through the enhanced appreciation of the hidden beauties of mathematics.

5 Taking Advantage of the Pedagogical Content of the Exhibits

If these exhibitions and their contents constitute a quality media tool, it would be disappointing if we could not use them also for more advanced educational purposes. Indeed, it would be a shame if the entire content of our collection, with a large pedagogical potential, remained asleep in some dark dormitories.

We can only regret that the so-called popular annual neighborhood events–where we see some classes of small merry pupils stopping for one moment with their guide in front of tables with so-called educational, books, games and various objects–of so little use to the training of the mind. What do these day visitors learn of mathematics? We show polyhedrons, we make some simple tessellations. What could the spectator glancing at the icosi-thingumajig learn? What new knowledge of mathematics did he come away with his superficial visit?

The exhibitions, of course, do not escape the same criticism. It is then advisable to value the contents. We reach the public by presentations made in a relaxed atmosphere, in the unusual setting of an enlightened public in a setting containing some of the beautiful works which will be commented on.

The content of the presentation obviously depends on the public being addressed. Age, educational level, and audience reactions are important factors. There is no ready-made formula.

Of course the way the presentation is made is important. It should by no means be an academic presentation. It is better to be joyful and enthusiatic. The speaker communicates his or her surprise at a property or particular fact, showing philosophic seriousness in front of such or such property or general, universal fact, coming and going from an example to the other one in apparently different situations which illustrate this fact, creating the global vision of a theory within which the same fact joins, evoking the history and how it is connected with other theories. Let us be desire that the auditor leaves the room relaxed, but also with the understanding of a new general concept, the knowledge of a new particular fact. Then maybe he will have the impression to have reached the threshold of the prestigious universe of mathematics, to have crossed a door and made a first step, the very first step, humble and reserved, in this world which formerly frightened him, and which seems to him today simple and radiant. Hopefully he will no longer fear mathematics, and will share this feeeling with others.

A large audience will accept and be interested in the main facts, in the main ideas and concepts behind the works, among which, above all, the concept of stability which has not yet been quite well understood. The presentation of these concepts should include a few words about their history, the extent of their incarnation, their importance to the physical, mathematical, artistic and philosophical worlds, and when possible some easy and fast explanations as well. Given in a favorable environment, these talks give rise to exchanges between the enthusiastic speaker and the audience, in a relaxed and rather joyful atmosphere.

The contents of these presentations at which it is hinted here will very likely arouse some reserves on behalf of the mathematicians. For the professionals, a good mathematician is the one who shows his community new properties, and who explains the reasons for their presence, who gives the proofs. The value of a researcher lies in creating new concepts and theories, in discovering new properties.

To see properties demands a familiarity and an attention which the public cannot acquire alone and be self-sufficient in three quarters of an hour. However, a speaker can very well during this short lapse of time succeed in awakening the attention of his public, and in guiding it to understand significant properties of the objects which it could not distinguish at first sight. So low it is, the presentation so conceived in the presence of works of art possesses an educational quality which we cannot underestimate. Still it is necessary to have warned the public beforehand about this fundamental aspect of the work of the mathematician.

We can only wonder that it was necessary to wait for year 1742 to notice that any even number is the sum of two prime numbers. Is it possible that the great Greek mathematicians had not realized this elementary fact? It happens that we pass next to the beauty which lines our way without becoming aware of it, and that the simplest things also are the most difficult to understand and to justify (prove). The mathematician who is able to prove the greater part of Goldbach's observation is destined for fame.

The second quality of the mathematician is to know how to give the irrefutable proof of the existence of a property. A certain dose of trick, but

also the familiarity with the various already used technics of demonstration, their control, the knowledge of numerous already well established properties are necessary to well lead new demonstrations. It is the reason why the professionals grant a large importance for any kind of exercise which develops the mental agility. And besides, an understanding of the demonstrations reveals a thorough knowledge of objects, generally facilitating the discovery of up to now unobserved properties.

The presentations address publics which can be qualified as virgin in mathematical subjects for the most part. Is it then reasonable to make detailed presentations before them? In these conditions, is the presentation, seen from the point of view of the professional mathematician, interesting enough to encourage a more widespread audience?

These last comments deserve to be qualified, because they are the expression of a somewhat extremist position. We know well that except for some very rare specialists, to go into detail the knowledge of the complete proofs of the ex-conjectures by Fermat or Poincaré is a matter of illusion. Happy already the one who can know the main lines of these demonstrations, the reasons which led to choose such or such mathematical tool, some of the progress which uncorked in the obtaining of results appearing now particular, intermediaries, but interesting in themselves. In front of the scale of certain demonstrations, the mathematician eventually eventually is satisfied with the knowledge and with the understanding brought by the datum of these guidelines.

During the maths and arts lectures, it can occur, in some rare and simple cases, and in front of certain public, that we can justify the assertion by the reasoning and the deduction, even give these indications evoked in the previous paragraph on the procedure of demonstration. But more commonly and more modestly within the framework of these presentations, it will be possible to mention the theories which come into play, their history and the subject of which they take care, their objectives, and especially to refer to physical, natural facts, these theories of which, through their statements, develop the representation and their effects.

After these presentations, the audience will therefore perhaps enrich their vocabulary with some mathematical terms denoting particular objects they have seen, or which have touched their eyes through beautiful images. These objects will have shown them some features and unexpected properties, objects of which they have in theory registered some fundamental concepts, objects inserted in theories of which the audience would have learned some fundamental concepts.

Would this optimistic comment be that one of a dreamer, even a humorist? We might think that at first, but doubt it in the light of the successful experiments made in France [2] in 2011 (Fig. 3) and in Greece [3], respectively in 2007 and

2012 (Fig. 1 et 2) with several classes of children ranging from 5 to 20 years of age.

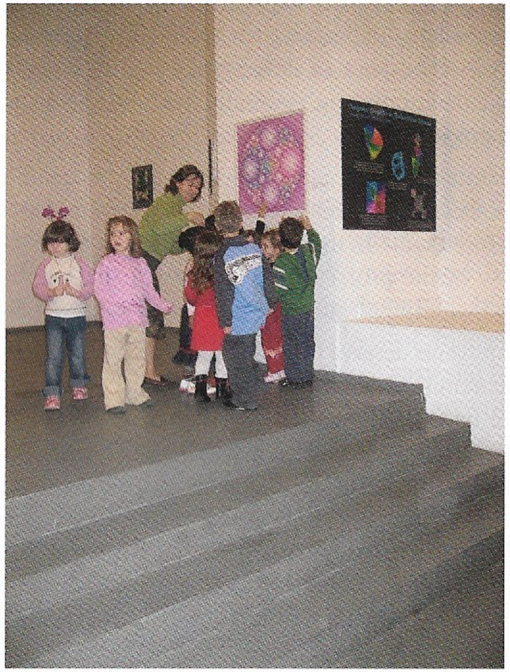

Fig. 1

Fig. 2

Fig. 3

It would be necessary, of course, to multiply them, and especially to enrich them. Local exhibits and exposés could be set up anywhere. With the permission of the authors, and perhaps a small payment to them, ESMA could send by e-mail the images of the works that the local organizer would like to show, to use, and then to print.

One can find on our website examples of such exposés, unfortunately in French, looking at *Bonne Année* |Part I|Part II and *Pâtisserie Mathématique* |Part I|PartII|Part III|Part IV. (cf. http://www.math-art.eu/Documents/ListOfAuthors-Publications(3).php#20).

6 Conclusion

To conclude, we shall resume simply the main part of these contributions.

The exhibitions, which attract the visitors by the novelties that they can discover, by the large diversity of the works and by the beauties that they reveal, allow the public to approach the field of mathematics in a relaxed and smoothing atmosphere. Often seduced by this unexpected world, here are finally our fellow countrymen pleasantly at peace with one of the most elegant sciences.

Presentations complete the exhibitions. They bring to the visitors a first knowledge of the concepts and mathematical facts which are developed in theories. Further, they allow to appreciate the interest, the importance of the concepts which come from their relevance and the generality of their embodiment. Then placed in the basis of the theories, their genesis is intimately connected to the history of the development of mathematics. Thus these presentations, through the marked initiatory character in the field of the mathematics, where the mathematics conjugate with the arts, contribute to promote thought.

Through the open-mindedness which they bring to the intellectual world and more particularly to the contemporary scientific world, these exhibitions and presentations play a positive and original role in the insertion of the individuals within our societies.

References

[1] C. P. Bruter, *Comprendre les mathématiques*, Odile Jacob, Paris, 1996.

[2] C. P. Bruter, "L'œuvre d'art mathématique, un outil pédagogique efficace", http://www.math-art.eu/Documents/pdfs/L'oeuvre_d'art,_outil.pdf.

[3] C. P. Bruter, *Matematika* (Mathématique et Art), *A'Eukleidnz*, 85, 2012, 1-4, et 88, 2012, 1-4. Texte établi par Dimosthenis Varopoulos d'après *Pâtisserie Mathématique* |Part I|PartII|Part III|Part IV. Cf. http://www.math-art.eu/Documents/ListOfAuthors-Publications(3).php#20.

Claude Paul Bruter
Retired Professor (Université Paris 12)
ESMA, c/o Institut Henri Poincaré
11 rue P. & M. Curie
75321 Paris Cedex 05
bruter@u-pec.fr