CARON'S WOODEN MATHEMATICAL OBJECTS

Abstract. We pay tribute to the mathematician-artist Joseph Caron by showing his collection of wooden models stored in the Institute Henri Poincaré in Paris.

1. THE MATHEMATICIAN

The collection of mathematical objects of the Institute Henri Poincaré in Paris includes in particular a series of wooden models made by Joseph Caron.

Joseph Caron 1849-1924 (photo courtesy of the ENS-Ulm library)

Joseph Caron entered the École Normale Supérieure in Paris in 1868. He was appointed professor of descriptive geometry at several parisian Lycées starting from 1871. The next year, according to the wish of Gaston Darboux, he was designated as director of graphical works at École Normale Supérieure. He wrote several handbooks of descriptive geometry ([2], [3], [4]). His

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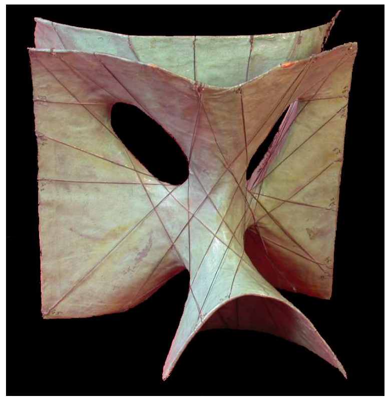
acute sense of geometric reality, combined with an interest in practical realizations, stimulated the students amongst them the young Henri Lebesgue whose taste turned therefore towards geometrical constructions. Actually, Caron produced physical models of geometrical objects stemming from the exercices and lectures by Darboux on curves and surfaces (see [5]), and so doing, he supplied the Cabinet de mathématiques (a kind of Cabinet of curiosities for mathematical beings directed by Darboux) at the Sorbonne.

Cabinet de mathématiques at the Sorbonne (photo courtesy of the Institute Henri Poincaré)

From 1872 to january 1915 (the first World War possibly stopped the work) he produced more than 80 models mainly in wood. Most of them are signed, however some of the first ones are not, maybe because nobody made him aware of the future history of science. For instance, in 1880 he wrote an article entitled «*Sur l'épure des 27 droites d'une surface du troisième degré dans le cas où ses droites sont réelles*» in the Bulletin de la Société Mathématique de France, and he is most likely (this is a conjecture) the author of the model herebelow (Man Ray took a picture of it in the 30's).



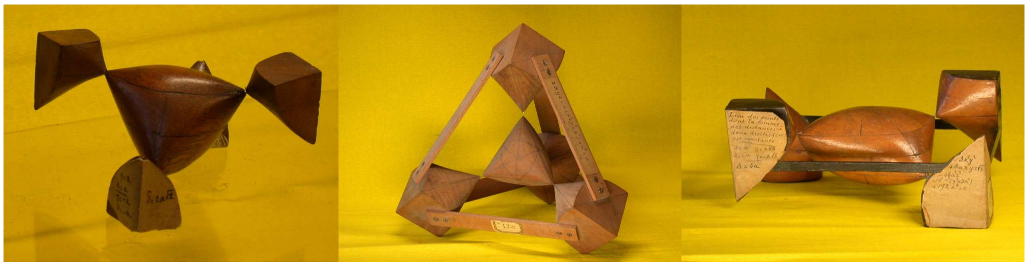
Cubic surface with 27 real lines (IHP collection)

In this paper, we shall focus on wooden models, and particularly on a series of eight models illustrating a point in optical theory.

2. SELECTION OF WOODEN MODELS

Here below we present a selection of wooden models made by Caron between 1910 and 1915 (IHP collection pictures).

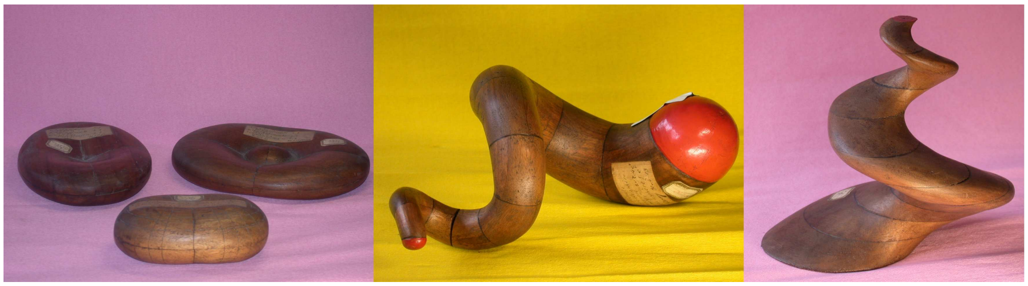
From left to right: Rational algebraic surface of degree eight generated by the plane section of a cylinder rolling on another cylinder. Rational algebraic surface of degree ten. Envelope of the normals for a hyperbolic paraboloid.



From left to right: Algebraic surface of degree four defined by the set of points whose sum of the distances to two lines is constant. Algebraic surface of degree three with tetrahedral symmetry. Algebraic surface of degree four defined by the set of points whose sum of the distances to two lines is constant.

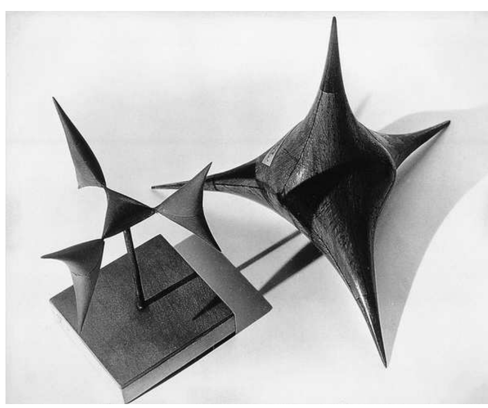
Three algebraic surfaces of degree four.

From left to right: Envelope of the normals for a Plücker conoid. Algebraic surface of degree four. Kummer surface with twelve real double points.



From left to right: Set of three deformations of an ellipsoid. Two spiral surfaces generated by circles.

From left to right: Henneberg's minimal surface. Rational surface of degree four.

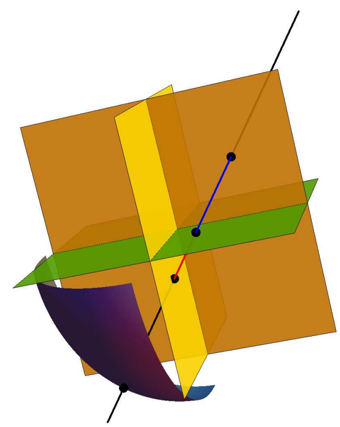


Two algebraic surfaces of degree four (after a photography by Man Ray, Cahiers d'Art, n°1-2, 1936).

This last picture (above) requires an explanation. The left-hand model is the same as the right one shown four figures above. The point is that the right-hand model has been lost. Thanks to Man Ray who photographed both models in 1935, we have a record of its existence.

3. A series of eight models made between 1912 and 1914.

The problem stated by Darboux is as follows: find the surfaces orthogonal to the lines three points of which of mutually constant distances moving on three orthogonal planes.

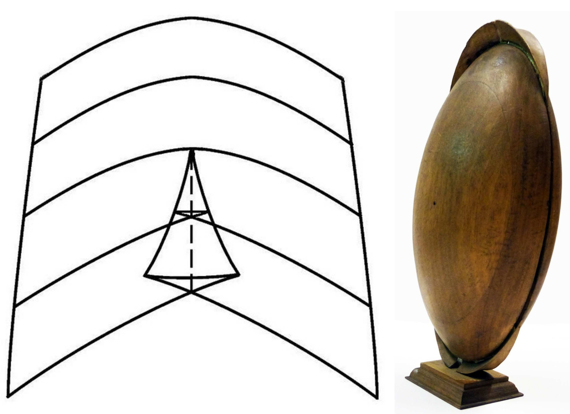


The surface (violet) is orthogonal to the moving line. The three orthogonal planes cut two segments (blue and red) of constant lengths on the line).

It takes its origin in optical theory: find a new geometric definition of a wave front following the work of Malus, Dupin, Niven,.... A wave front is the surface generated at the time t fixed, by the electromagnetic particules emitted by a body at time t_0. Therefore it is orthogonal to the rays of particules. Such a wave front generates a caustic where the energy is concentrated. It is a surface tangent to all the rays of particules. The eight Caron's models below illustrate these notions.

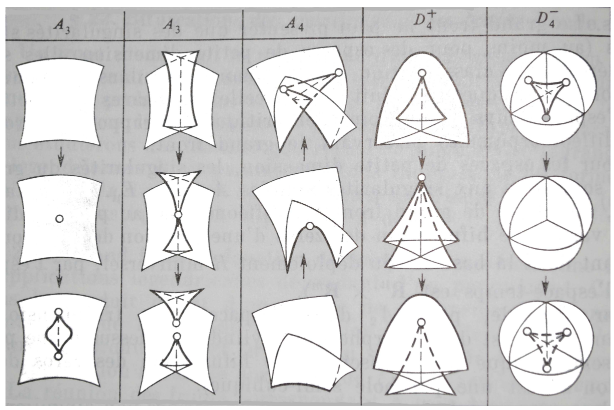
(three left-hand columns) Wave front at six different values of the time. (rightmost column) Caustics.

While time is varying, the shape of the wave front changes smoothly, but sometimes singularities pop up, that is points where the curvature is no longer bounded, like on a cuspidal edge or a swallowtail.



(left) Swallowtail at the cusp of a cuspidal edge. (right) Four swallowtails on the Caron's model.

At certain particular values of the time, a singularity not existing before, suddenly appears. The shape of the wave front changes drastically. Such a phenomenon is called a bifurcation or a metamorphosis. Between two metamorphoses, either the global shape remains smoothly stable, like a sphere deformed by small hollows or bumps, or self-intersection occurs or is recomposed by surgery. The key point to understand the evolution of the wave front consists therefore in describing self-intersections and metamorphoses. In a generic wave front, the metamorphoses have been classified by V.I. Arnol'd in 1974 ([1]). There are five types shown below.



The five metamorphoses of a generic one-parameter wave front (excerpt from [1]).

It is noticeable that the Caron's models are precisely chosen in order to suggest the transitions between nonequivalent shapes as shown below.

Swallowtails hyperbolic confluence (type A_3).

Double swallowtail (type D_4^+).

Similarly, looking at other pairings of models, we can recognize a quadruple point, the birth of a self-intersection curve, and the birth of two swallowtails (type A_3).

4. THE ARTIST

The surrealists never mentionned the name of Caron, they gave him no credit for his work even though most of the models they were using were signed. Of course Caron was dead, but today we can't imagine applying such a treatment to Dali, Ernst, Man Ray and others. Maybe, Caron did not consider himself as an artist, however his wooden models are nicely finished, the wood is polished and varnished, the models are attached on structures to be presented on supports. And last but not least, the surrealists touch them and insert them in their own productions. That is a kind of definition of a piece of art. As a consequence we should consider Joseph Caron as a mathematician-artist.

References

[1] V.I. Arnol'd, *Méthodes mathématiques de la mécanique classique*, Mir 1976 (traduction de l'édition russe de 1974).

[2] J. Caron, *Cours de géométrie descriptive (droite et plan). À l'usage des aspirants au baccalauréat ès-sciences*, librairie Germer Baillière et C^ie^, Paris 1882.

[3] J. Caron, *Cours de géométrie descriptive. À l'usage des classes de mathématiques spéciales*, librairie E. Foucart, Paris 1883.

[4] J. Caron, *Cours de géométrie descriptive. Géométrie cotée. À l'usage des candidats à l'École Spéciale Militaire de Saint-Cyr*, librairie Germer Baillière et C^ie^, Paris 1898.

[5] G. Darboux, *Leçons sur la théorie générale des surfaces*, première partie, Gauthier-Villars, Paris (1914).

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